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THE ANTIPLANE PROBLEM OF ELECTROELASTICITY FOR A CYLINDER WITH A TUNNEL CRACK EXCITED BY AN ARBITRARY SYSTEM OF SURFACE ELECTRODES†

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The antiplane mixed boundary-value problem of electroelasticity of the oscillations of an infinite piezoceramic cylinder, weakened by a curvilinear tunnel crack, is considered. Using special integral representations of the solution, the boundary-value problem is reduced to a system of singular integro-differential equations of the second kind with discontinuous kernels. The results of a numerical realization of the algorithm, characterizing the amplitude-frequency characteristics of a piecewise-uniform cylinder and the behaviour of the components of the electroelastic field in the region and on the boundary of the cylinder under conditions of the inverse piezoelectric effect, are presented. © 2002 Elsevier Science Ltd. All rights reserved.

The static and dynamic antiplane problems of electroelasticity for a circular cylinder with one and two symmetrically placed electrodes have been investigated by the series method [1]. The oscillations of infinite piezoceramic cylinders with tunnel-crack type defects and linear rigid inclusions have been considered by the method of singular integral equations for the direct piezoelectric effect in [2].

1. FORMULATION OF THE PROBLEM

Consider a piezoceramic cylinder containing a curvilinear tunnel crack (a cut) L, referred to a Cartesian system of coordinates $Ox_1x_2x_3$, infinite in the direction of the axis of symmetry of the material x_3. On the surface of the cylinder, free from mechanical forces, there are $2n$ thin electrodes, infinitely long in the direction of the x_3 axis, with specified electric potential differences. The non-electrode parts of the cylinder surface have an interface with a vacuum (air). The boundaries of the k-th electrode are defined by the quantities β_{2k-1} and β_{2k} $(k = 1, 2, \ldots, 2n)$, while the electric potential on them is given by the quantity $\phi_k^* = \mathrm{Re}(\Phi_k^* e^{-i\omega t})$ (ω is the angular frequency and t is the time). It is assumed that the curvature of the contours L and C are Hölder-class functions [3], and the electrodes are ideally conducting and absolutely flexible.

Under these conditions, an electroelastic field is set up in the piecewise-uniform cylinder, corresponding to the state of antiplane deformation [1]. The complete system of differential equations in the quasistatic approximation includes the following relations

$$\partial_1\sigma_{13} + \partial_2\sigma_{23} = \rho\frac{\partial^2 u_3}{\partial t^2}, \quad \partial_i = \frac{\partial}{\partial x_i} \tag{1.1}$$

$$\begin{aligned} &\sigma_{m3} = c_{44}^E \partial_m u_3 - e_{15}E_m \\ &D_m = e_{15}\partial_m u_3 + э_{11}^\varepsilon E_m \quad (m = 1, 2) \end{aligned} \tag{1.2}$$

$$\begin{aligned} &\operatorname{div}\mathbf{D} = 0 \\ &\mathbf{E} = -\operatorname{grad}\phi \end{aligned} \tag{1.3}$$

Here (1.1) is the equation of motion, (1.2) are the material equations of the medium, (1.3) are the equations of electrostatics [4], σ_{m3} $(m = 1, 2)$ are the components of the mechanical stress tensor, u_3 is the component of the elastic displacement vector in a direction parallel to the cylinder axis, $\mathbf{E}$ and $\mathbf{D}$ are the electric field and electric induction vectors, ϕ is the electric potential, c_{44}^E is the shear modulus,

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measured at a constant value of the electric field, e_{15} is the piezoelectric constant, $\ni_{11}^{\varepsilon}$ is the permittivity, measured at fixed deformations respectively, and ρ is the mass density of the material.

We reduce the system of equations (1.1)–(1.3) to differential equations in the displacement u_3 and the electric potential ϕ

$$c_{44}^{E}\nabla^2 u_3 + e_{15}\nabla^2\phi = \rho\frac{\partial^2 u_3}{\partial t^2}, \quad e_{15}\nabla^2 u_3 - \ni_{11}^{\varepsilon}\nabla^2\phi = 0 \tag{1.4}$$

From Eqs (1.4) we have the relations

$$\begin{gathered}\nabla^2 u_3 - c^{-2}\frac{\partial^2 u_3}{\partial t^2} = 0, \quad \nabla^2 F = 0 \\ \phi = \frac{e_{15}}{\ni_{11}^{\varepsilon}}u_3 + F, \quad c = \sqrt{\frac{c_{44}^{E}\left(1+k_{15}^2\right)}{\rho}}, \quad k_{15} = \frac{e_{15}}{\sqrt{c_{44}^{E}\ni_{11}^{\varepsilon}}}\end{gathered} \tag{1.5}$$

where c is the velocity of a shear wave in the piezoceramic medium and k_{15} is the electromechanical coupling coefficient [1].

Taking relations (1.2), (1.3) and (1.5) into account, the mechanical and electrical quantities can be expressed in terms of the functions u_3 and F using the formulae

$$\begin{gathered}\sigma_{13} - i\sigma_{23} = 2\frac{\partial}{\partial z}\left[c_{44}^{E}\left(1+k_{15}^2\right)u_3 + e_{15}F\right] \\ D_1 - iD_2 = -2\ni_{11}^{\varepsilon}\frac{\partial F}{\partial z}, \quad E_1 - iE_2 = -2\frac{\partial}{\partial z}\left(F + \frac{e_{15}}{\ni_{11}^{\varepsilon}}u_3\right), \quad z = x_1 + ix_2\end{gathered} \tag{1.6}$$

Assuming

$$u_3 = \operatorname{Re}\left(U_3 e^{-i\omega t}\right), \quad \phi = \operatorname{Re}\left(\Phi e^{-i\omega t}\right), \quad F = \operatorname{Re}\left(e^{-i\omega t}F^*\right)$$

we can write Eqs (1.5) in terms of the amplitude quantities

$$\nabla^2 U_3 + \gamma^2 U_3 = 0, \quad \nabla^2 F^* = 0, \quad \Phi = \frac{e_{15}}{\ni_{11}^{\varepsilon}}U_3 + F^*; \quad \gamma = \frac{\omega}{c} \tag{1.7}$$

where γ is the wave number.

Assuming that the crack edges are free from mechanical stresses, we can represent the mechanical and electrical boundary conditions on the contour L as follows [1]:

$$\left(\sigma_{13}\cos\psi + \sigma_{23}\sin\psi\right)^{\pm} = 0 \tag{1.8}$$

$$E_s^+ = E_s^-, \quad D_n^+ = D_n^- \tag{1.9}$$

Here E_s is the tangential component of the electric field vector, D_n is the normal component of the electric induction vector, ψ is the angle between the normal to the left edge of the cut L and the Ox_1 axis, and the plus and minus signs refer to the left and right edges of the cut when moving from its origin a to the end b (Fig. 1). Conditions (1.9) express the fact that the corresponding components of the electric field do not undergo jumps on passing through the cut L.

Taking representations (1.5) and (1.6) into account, the boundary conditions on the surfaces of the cylinder and the crack can be represented in the form

$$\begin{gathered}\frac{\partial}{\partial n}\left\{c_{44}^{E}\left(1+k_{15}^2\right)U_3 + e_{15}F^*\right\} = 0 \text{ on } C; \quad F^* + \frac{e_{15}}{\ni_{11}^{\varepsilon}}U_3 = \Phi^*\left(\zeta^*\right), \quad \zeta^* \in C_\phi \\ D_n^* = -\ni_{11}^{\varepsilon}\frac{\partial F^*}{\partial n} = 0 \text{ on } C \setminus C_\phi; \quad \frac{\partial}{\partial n}\left\{c_{44}^{E}\left(1+k_{15}^2\right)U_3 + e_{15}F^*\right\}^{\pm} = 0 \text{ on } L\end{gathered} \tag{1.10}$$

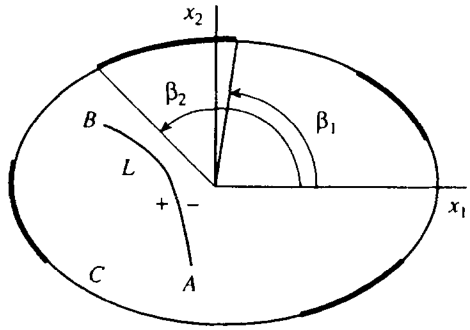

Fig. 1

where C_ϕ is the part of the contour C corresponding to the electroded surface of the cylinder, the operator $\partial/\partial n$ denotes a derivative along the normal to the boundary contour, and the asterisks denote the amplitudes of the corresponding quantities.

Hence, the problem consists of determining the functions U_3 and F^* from Eqs (1.7), the boundary conditions (1.10) and the electrical conditions (1.9) on the cut L.

2. THE INTEGRO-DIFFERENTIAL EQUATIONS OF THE BOUNDARY-VALUE PROBLEM OF ELECTROELASTICITY

Following the approach described previously [2], we will represent the amplitudes of the required functions in the form

$$U_3(z) = \int_C p(\zeta^*) H_0^{(1)}(\gamma r_1) ds - \frac{i}{4} \int_L [U_3] \frac{\partial H_0^{(1)}(\gamma r)}{\partial n_\zeta} ds$$
$$F^*(z) = \int_C f(\zeta^*) \frac{\partial}{\partial n_{\zeta^*}} \ln r_1 ds + \frac{e_{15}}{2\pi \varepsilon_{11}^\varepsilon} \int_L \left[\frac{dU_3}{ds}\right] \arg(z-\zeta) ds \qquad (2.1)$$
$$r = |z-\zeta|, \quad r_1 = |\zeta^* - z|, \quad \zeta \in L, \quad \zeta^* \in C$$

Here $H_\nu^{(1)}(x)$ is the Hankel function of the first kind of order ν and ds is an element of length of the arc of the contour along which the integration is carried out.

Integral representations (2.1) satisfy the differential equations (1.7), ensure that there is a jump in the displacement and continuity of the stress vector on L, and also ensure that the electrical conditions (1.9) are automatically satisfied.

Substituting the limiting values of the functions (2.1) and their derivatives as $z \to \zeta_0 \in L$ and $z \to \zeta_0^* \in C$ into boundary conditions (1.10), we arrive at a system of singular integro-differential equations of the second kind

$$-2ip(\zeta_0^*) + \int_C p(\zeta^*) g_1(\zeta^*, \zeta_0^*) ds + \int_C f'(\zeta^*) g_2(\zeta^*, \zeta_0^*) ds +$$
$$+ \int_L \left[\frac{dU_3}{ds}\right] g_3(\zeta, \zeta_0^*) ds + \int_L [U_3] g_4(\zeta, \zeta_0^*) ds = 0$$
$$\pi f(\zeta_0^*) + \int_C p(\zeta^*) g_5(\zeta^*, \zeta_0^*) ds + \int_C f(\zeta^*) g_6(\zeta^*, \zeta_0^*) ds + \qquad (2.2)$$
$$+ \int_C \left[\frac{dU_3}{ds}\right] g_7(\zeta, \zeta_0^*) ds + \int_C [U_3] g_8(\zeta, \zeta_0^*) ds = \Phi^*(\zeta_0^*), \quad \zeta_0^* \in C_\phi$$
$$\int_C f'(\zeta^*) g_9(\zeta^*, \zeta_0^*) ds + \int_L \left[\frac{dU_3}{ds}\right] g_{10}(\zeta, \zeta_0^*) ds = 0, \quad \zeta_0^* \in C \setminus C_\phi$$

$$\int_C p(\zeta^*) g_{11}(\zeta^*, \zeta_0) ds + \int_C f'(\zeta^*) g_{12}(\zeta^*, \zeta_0) ds +$$

$$+ \int_L \left[\frac{dU_3}{ds}\right] g_{13}(\zeta, \zeta_0) ds + \int_L [U_3] g_{14}(\zeta, \zeta_0) ds = 0$$

in which the kernels g_m ($m = 1, 2, \ldots, 14$) are defined by the expressions

$$g_1(\zeta^*, \zeta_0^*) = \frac{2}{\pi i} \mathrm{Re} \frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*} + \gamma H_1(\gamma r_{10}) \cos(\psi_{10} - \alpha_{10})$$

$$g_2(\zeta^*, \zeta_0^*) = \frac{e_{15}}{c_{44}^E (1 + k_{15}^2)} \mathrm{Im} \frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*}, \quad g_3(\zeta, \zeta_0^*) = \frac{1}{2\pi(1 + k_{15}^2)} \mathrm{Im} \frac{e^{i\psi_{10}}}{\zeta - \zeta_0^*}$$

$$g_4(\zeta, \zeta_0^*) = \frac{i\gamma^2}{8} \left[H_2(\gamma r_{20}) \cos(\psi + \psi_{10} - 2\alpha_{20}) - H_0^{(1)}(\gamma r_{20}) \cos(\psi - \psi_{10})\right]$$

$$g_5(\zeta^*, \zeta_0^*) = \frac{e_{15}}{\text{Э}_{11}^\varepsilon} H_0^{(1)}(\gamma r_{10}), \quad g_6(\zeta^*, \zeta_0^*) = \mathrm{Re} \frac{e^{i\psi_1}}{\zeta^* - \zeta_0^*}$$

$$g_7(\zeta, \zeta_0^*) = \frac{e_{15}}{2\pi \text{Э}_{11}^\varepsilon} \alpha_{20}, \quad g_8(\zeta, \zeta_0^*) = -\frac{i e_{15}}{4 \text{Э}_{11}^\varepsilon} \gamma H_1^{(1)}(\gamma r_{20}) \cos(\psi - \alpha_{20})$$

$$g_9(\zeta^*, \zeta_0^*) = \mathrm{Im} \frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*}, \quad g_{10}(\zeta, \zeta_0^*) = -\frac{e_{15}}{2\pi \text{Э}_{11}^\varepsilon} \mathrm{Im} \frac{e^{i\psi_{10}}}{\zeta - \zeta_0^*}$$

$$g_{11}(\zeta^*, \zeta_0) = \frac{2}{\pi i} \mathrm{Re} \frac{e^{i\psi_0}}{\zeta^* - \zeta_0} + \gamma H_1(\gamma r_{30}) \cos(\psi_0 - \alpha_{30})$$

$$g_{12}(\zeta^*, \zeta_0) = \frac{e_{15}}{c_{44}^E (1 + k_{15}^2)} \mathrm{Im} \frac{e^{i\psi_0}}{\zeta^* - \zeta_0}, \quad g_{13}(\zeta, \zeta_0) = \frac{1}{2\pi(1 + k_{15}^2)} \mathrm{Im} \frac{e^{i\psi_0}}{\zeta - \zeta_0}$$

$$g_{14}(\zeta, \zeta_0) = \frac{i\gamma^2}{8} \left[H_2(\gamma r_0) \cos(\psi + \psi_0 - 2\alpha_0) - H_0^{(1)}(\gamma r_0) \cos(\psi - \psi_0)\right]$$

$$H_1(x) = \frac{2i}{\pi x} + H_1^{(1)}(x), \quad H_2(x) = \frac{4i}{\pi x^2} + H_2^{(1)}(x)$$

$$r_0 = |\zeta_0 - \zeta|, \quad \alpha_0 = \arg(\zeta_0 - \zeta), \quad r_{10} = |\zeta^* - \zeta_0^*|, \quad \alpha_{10} = \arg(\zeta^* - \zeta_0^*)$$

$$r_{20} = |\zeta_0^* - \zeta|, \quad \alpha_{20} = \arg(\zeta_0^* - \zeta), \quad r_{30} = |\zeta^* - \zeta_0|, \quad \alpha_{30} = \arg(\zeta^* - \zeta_0)$$

$$\psi = \psi(\zeta), \quad \psi_1 = \psi(\zeta^*), \quad \psi_0 = \psi(\zeta_0), \quad \psi_{10} = \psi(\zeta_0^*), \quad \zeta, \zeta_0 \in L, \quad \zeta^*, \zeta_0^* \in C$$

Here ψ and ψ_1 are the angles between the normals to the contours L and C and the Ox_1 axis respectively and $\Phi(\zeta_0^*)$ is a piecewise-constant function, which specifies the values of the amplitudes of the electrical potentials on the electrodes.

For system (2.2) to be uniquely solvable in the class of functions with derivatives that are unbounded close to the ends of the cut L [3], it is necessary to consider it together with the additional condition

$$\int_L \left[\frac{dU_3}{ds}\right] ds = 0 \tag{2.3}$$

which expresses the fact that the displacement jumps at the vertices of the cut L are equal to zero. Moreover, condition (2.3) ensures that the integral representation of the function $F^*(z)$ in (2.1) is unique.

By determining the functions $[U_3]$, $p(\zeta^*)$ and $f(\zeta^*)$ from system (2.2), by using formulae (1.5) and (1.6) together with representations (2.1) we can determine all the components of the electroelastic field in the piecewise-uniform cylinder. Note also that a solution of system (2.2) together with condition (2.3) exists for any frequency ω not identical with the natural frequency.

Introducing parametrization of the contour C using the equations $\zeta^* = \zeta^*(\beta)$, $\zeta_0^* = \zeta^*(\beta_0)$ $(0 \leq \beta, \beta_0 \leq 2\pi)$, we obtain an expression for the amplitude of the electric charge distribution density $q_k(\beta)$ on the k-th electrode. Bearing in mind that the cylinder is in contact with a vacuum, we can write

$$q_k(\beta) = D_n^{(k)*}(\beta), \quad \beta_{2k-1} < \beta < \beta_{2k} \tag{2.4}$$

Here $D_n^{(k)*}(\beta)$ is the amplitude of the normal component of the electric induction vector on the part of the cylinder surface covered with the k-th electrode.

Making use of integral representation (2.1) for the function $F^*(z)$ and taking into account Eqs (2.4) and (1.6) we obtain

$$q_k(\beta_0) = -\text{э}_{11}^{\varepsilon} \int_C f'(\zeta^*) \operatorname{Im} \frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*} ds, \quad \zeta_0^* \in C_{\Phi_k} \tag{2.5}$$

where C_k is the part of the contour C on which the k-th electrode is situated.

Integrating expression (2.5) with respect to the variable β_0 in the limits from β_{2k-1} to β_{2k}, we obtain the amplitude value of the total charge Q_k of the k-th electrode, referred to unit length of the electrode. The current through this electrode that is equal to the conduction current in the generator circuit, can be found from the formula

$$I_k(t) = \operatorname{Re}\left\{ i\omega e^{-i\omega t} \int_{\beta_{2k-1}}^{\beta_{2k}} q_k(\beta_0) s'(\beta_0) d\beta_0 \right\}, \quad s'(\beta_0) = \frac{ds}{d\beta_0} \tag{2.6}$$

Equation (2.6) enables us to determine the antiresonance frequencies for which $I_k(t) = 0$.

Note that in the case of antiplane deformation, the longitudinal shear stresses on the surface free from mechanical load do not have singularities at the edges of the electrodes [5]. Nevertheless, the components of the electric induction vector possess root-type singularities at the edges of the electrodes, which follows directly from an asymptotic analysis of singular integral equations (2.2) and expressions (2.4) and (2.5).

3. THE STRESS INTENSITY FACTOR

To calculate the stress intensity factor K_{III} [6] we will obtain the principal asymptotic form of the shear stress on a crack which extends to the vertex. Here we will start from formulae which define the behaviour of the Cauchy-type integrals in the neighbourhood of the ends of the cut L in the case when the density has a power singularity [7],

$$\frac{1}{2\pi i} \int_L \frac{\tau(\zeta) d\zeta}{\zeta - z} = \begin{cases} \dfrac{e^{\sigma\pi i} \tau_0(a)(z-a)^{-\sigma}}{2i \sin \pi\sigma} + \Lambda_1(z), & z \in O(a) \\ -\dfrac{e^{-\sigma\pi i} \tau_0(b)(z-b)^{-\sigma}}{2i \sin \pi\sigma} + \Lambda_2(z), & z \in O(b) \end{cases} \tag{3.1}$$

$$\tau(\zeta) = \frac{\tau_0(\zeta)}{(z-c)^{\sigma}}, \quad \sigma = \varkappa_1 + i\varkappa_2, \quad 0 \leq \varkappa_1 \leq 1$$

The following relations hold for the functions $\Lambda_m(z)$

$$\lim_{z \to a} \Lambda_1(z)(z-a)^{\sigma} = 0, \quad \lim_{z \to b} \Lambda_2(z)(z-b)^{\sigma} = 0$$

It follows from an asymptotic analysis of the last singular integro-differential equation in (2.2) in the neighbourhood of the vertex of the cut L that $\sigma = 1/2$. Hence, introducing parametrization of the crack contour $\zeta = \zeta(\delta)$, we can put

$$\left[\frac{dU_3}{ds}\right] = \frac{\Omega_0(\delta)}{s'(\delta)\sqrt{1-\delta^2}}, \quad s'(\delta) = \frac{ds}{d\delta} > 0, \quad -1 \leq \delta \leq 1 \tag{3.2}$$

where the function $\Omega_0(\delta)$ is Hölder continuous.

We have (retaining only terms which make a contribution to the asymptotic form)

$$\sigma_n = \mathrm{Re}(S_n e^{-i\omega t})$$
$$S_n = c_{44}^E \frac{\partial U_3}{\partial n} + \ldots = c_{44}^E \left[e^{i\psi_c} \frac{\partial U_3}{\partial z} + e^{-i\psi_c} \frac{\partial U_3}{\partial \bar{z}} \right] + \ldots \tag{3.3}$$

where c is the tip of the cut and $\psi_c = \psi(c)$.

On the basis of expressions (2.1) we can write the principal part of function (3.3)

$$S_n^0 = \frac{c_{44}^E}{2\pi} \int_L \left[\frac{dU_3}{ds} \right] \mathrm{Im} \frac{e^{i\psi_c}}{\zeta - z} ds \tag{3.4}$$

Using asymptotic formulae (3.1) and taking relations (3.2) into account we obtain

$$S_n^0 = \pm c_{44}^E \frac{\Omega_0(\pm 1)}{2\sqrt{2r^* s'(\pm 1)}}, \quad s'(\pm 1) = \left. \frac{ds}{d\delta} \right|_{\delta = \pm 1} \tag{3.5}$$

Here $r^* = |z - c|$, the lower sign relates to the tip $c = a$ and the upper sign relates to $c = b$.

Starting from relations (3.5) we can find the stress intensity factor

$$K_{\mathrm{III}}^{\pm} = \lim_{r^* \to 0} \sqrt{2\pi r^*} \sigma_n^0 = \pm \frac{c_{44}^E}{2} \sqrt{\frac{\pi}{s'(\pm 1)}} \mathrm{Re}[\Omega_0(\pm 1) e^{-i\omega t}] \tag{3.6}$$

The asymptotic form of the normal component of the electric induction vector on the extension beyond the tip of the cut is

$$D_n = D_1 \cos\psi(\pm 1) + D_2 \sin\psi(\pm 1) = \pm e_{15} \frac{\mathrm{Re}[\Omega_0(\pm 1) e^{-i\omega t}]}{2\sqrt{2r^* s'(\pm 1)}} \tag{3.7}$$

The remaining electrical quantities in the neighbourhood of the cut L are bounded. From the equations of state (1.2) we have

$$\sigma_n = c_{44}^E \frac{\partial u_3}{\partial n} - e_{15} E_n, \quad D_n = e_{15} \frac{\partial u_3}{\partial n} + э_{11}^{\varepsilon} E_n \tag{3.8}$$

where D_n is the normal component of the electric induction on the arc L', as close to L as desired. Since $[\sigma_n] = [D_n] = 0$ and the determinant of system (3.8) is non-zero, we obtain $[\partial u_3/\partial n] = [E_n] = 0$. Hence, we extend the electric field vector E continuously through the cut.

4. EXAMPLES

Consider a piezoceramic cylinder (made of PZT-4 material [1]) of elliptic cross-section, containing a rectilinear crack, oriented at an angle ϑ to the Ox_1 axis. The cylinder is excited by two electrodes with a difference in the amplitudes of the potentials of $2\Phi^*$, the centres of which lie on the x_2 axis. The parametric equations of the contours L and C respectively have the form

$$\zeta = l\delta e^{i\vartheta}, \quad \delta \in [-1, 1]$$
$$\zeta^* = R_1 \cos\beta + iR_2 \sin\beta, \quad \beta \in [0, 2\pi] \tag{4.1}$$

The system of integro-differential equations (2.2), together with condition (2.3) and taking Eqs (4.1) into account, was solved by a special scheme of the method of quadratures [2].

In Fig. 2 we show a graph of the relative stress intensity factor $\langle K_{\mathrm{III}}^{+} \rangle = c_{44}^E \sqrt{\pi l / s'(1)} \, |\Omega_0(1)| / (2e_{15}|\Phi^*|)$ and the quantity $Q^* = |Q/э_{11}^{\varepsilon}\Phi^*)|$ (Q is the amplitude of the total charge on the electrode) as a function of the normalized wave number γR with $R_1/R_2 = 1$, $l/R = 0.2$, $\vartheta = 0$ ($R = (R_1 + R_2)/2$, where $2l$ is the cut length. The values of the normalised wave numbers, corresponding to the first three

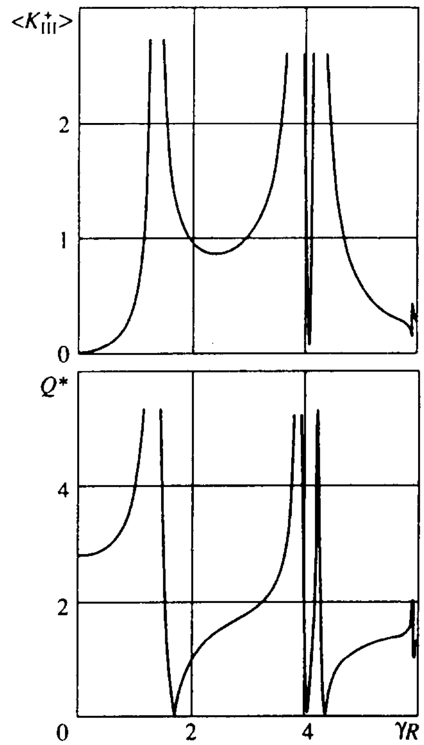

Fig. 2

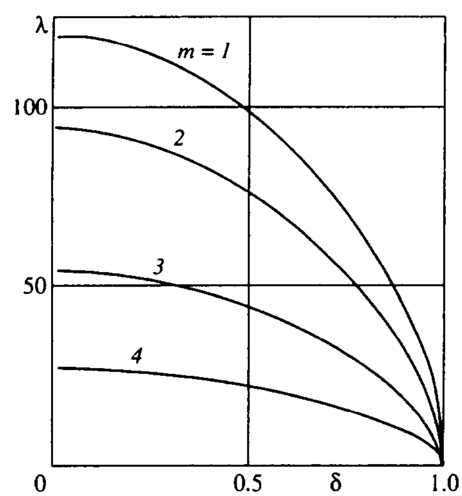

Fig. 3

natural frequencies of the oscillations in this case are such that $\gamma_{(1)}P \approx 1.35$, $\gamma_{(2)}P \approx 3.9$, $\gamma_{(3)}P \approx 4.24$. Knowing the value of $\langle K^{+}_{\mathrm{III}}\rangle$, the stress intensity factor can be determined from the formula

$$K^{\pm}_{\mathrm{III}} = \pm e_{15} \,|\, \Phi^* \,|\, \langle K^{\pm}_{\mathrm{III}} \rangle \cos\,(\omega t - \arg\Omega_0(\pm 1)) / \sqrt{l}$$

Figure 3 illustrates the change in the value of $\lambda = c^{E}_{44}[U_3]/|\Phi^*|$, which represents the jump in the displacement on the crack for different angles of orientation of the crack in a circular cylinder with $\gamma R = 3$ and $l/R = 0.4$ (in view of the symmetry, for $\delta \geqslant 0$). The curve with number m is drawn for the value $\vartheta = (m-1)\pi$. Note that when $\vartheta = \pi/2$ the presence of a crack does not give rise to perturbations in the electroelastic state of the cylinder.

In the versions of the calculation considered, the following values of the parameters defining the position of the electrodes were specified: $\beta_1 = 5\pi/14$, $\beta_2 = 9\pi/14$, $\beta_3 = 19\pi/14$, $\beta_4 = 23\pi/14$.

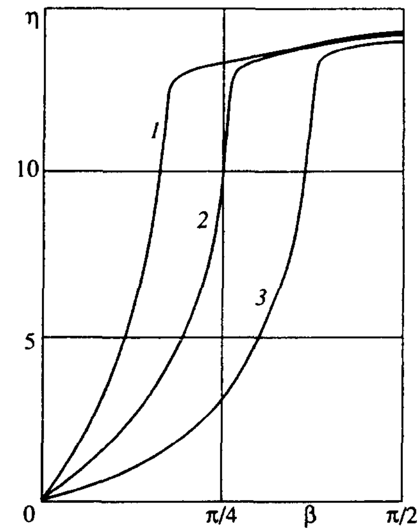

Fig. 4

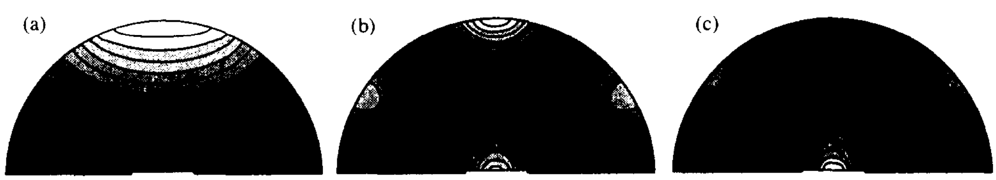

Fig. 5

In Fig. 4 we show data characterizing the distribution of the quantity $\eta = c_{44}^{E}\,|U_3/\Phi^*|$ on the contour of a piecewise-uniform cylinder for different areas of the electrode covering for $R_1/R_2 = 2$, $l/R_1 = 0.4$, $\vartheta = 0$ and $\gamma R = 0.5$ for the following values of the parameters: $\beta_1 = \pi/6$, $\beta_2 = 5\pi/6$, $\beta_3 = 7\pi/6$, $\beta_4 = 11\pi/6$ (curve 1), $\beta_k = (2k-1)\pi/4$ $(k = 1, 2, 3, 4)$ (curve 2) and $\beta_1 = 5\pi/14$, $\beta_2 = 9\pi/14$, $\beta_3 = 19\pi/14$, $\beta_4 = 23\pi/14$ (curve 3).

The level lines of the modulus of the displacement amplitude in a piecewise-uniform cylinder in the region of the first three natural frequencies of the oscillations for $l/R = 0.2$ are presented in Figs 5(a–c) (in view of the symmetry we only show half the section). The brightest zones correspond to maximum values of $|U_3|$. We assumed $\beta_1 = 5\pi/14$, $\beta_2 = 9\pi/14$, $\beta_3 = 19\pi/14$, $\beta_4 = 23\pi/14$ in the calculations.

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